ISOTROPIC CORRECTIONS TO THE MAXWELL DISTRIBUTION FUNCTIONS IN A PLASMA AND RATE OF ENERGY EXCHANGE

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The isotropic correction to the Maxwell electron distribution function in a two-temperature completely ionized plasma was obtained in [1] by the direct solution of the kinetic equation for electrons. In this article, such a correction is found by the direct solution of the kinetic equation for electrons. In this article, such a correction is found by the Chapman-Enskog method [2] applied to a two-temperature plasma [3]. This makes it possible to obtain corrections to the distribution functions for both electrons and heavy particles. The case of a partially ionized plasma is considered, and an expression is obtained for the rate of energy exchange between electrons and heavy particles for an arbitrary interaction law with allowance for the first corrections.

Let us represent the isotropic part of the distribution function in the form

$$f_\alpha = f_\alpha^\circ (1 - F_\alpha). \tag{1}$$

Here f_α° is the Maxwell distribution function for particles of type α and the subscript α = 1, 2, 3, respectively, for singly-charged ions, electrons, and neutrals. The electron temperature T_2 differs from the heavy particle temperature $T = T_1 = T_3$. Using the solution of [3], we obtain a system of equations for F_α:

$$\sum_{\beta=1}^{3} J_{\alpha\beta}(f_\alpha^\circ f_\beta^\circ) = I_\alpha(F_\alpha) \quad (\alpha = 1, 2, 3). \tag{2}$$

Integrals for $J_{\alpha\beta}$ and I_α are given in [3]. System (2) is solved in the form of a series expansion in Sonine polynomials $S_{1/2}^{(p)}(x)$, [2]:

$$F_\alpha = \sum_{r=2}^{\infty} g_{\alpha r} S_{1/2}^{(r)}(u_\alpha^2). \tag{3}$$

Here u_α is the dimensionless particle velocity in a coordinate system moving with the center of inertia of the mixture. The expansion begins with the second polynomial, so that the corrections do not affect the density n_α and temperature T_α. Leaving one polynomial in the expansion (3), we obtain a solution in the form

$$g_{12} = \frac{\psi_{12}}{c_{22}^{11}}\theta_1, \qquad g_{22} = \frac{\psi_{22}}{c_{22}^{22}}, \qquad g_{32}\ \frac{\psi_{32}}{c_{22}^{33}}\theta_3 \tag{4}$$

Here

$$\theta_1 = \frac{1 - (\psi_{32}c_{22}^{13} / \psi_{12}c_{22}^{33})}{1 - [(c_{22}^{13})^2 / c_{22}^{11}c_{22}^{33}]}, \qquad \theta_3 = \frac{1 - (\psi_{12}c_{22}^{31} / \psi_{32}c_{22}^{11})}{1 - [(c_{22}^{13})^2 / c_{22}^{11}c_{22}^{33}]},$$

$$\psi_{12} = \frac{9}{2}\frac{n_1}{\tau_2}\frac{T_2 - T}{T_2}\frac{m_2^2}{m^2},$$

$$\psi_{32} = \frac{n_2 n_3}{n_2 + n_3}\frac{T_2 - T}{T}\frac{m_2^2}{m^2}\frac{1}{\tau_{23}}\left(z_1\frac{T}{T_2} - z_2\frac{T_2 - T}{T}\right), \tag{5}$$

$$\psi_{22} = -\frac{T_2 - T}{T_2}\frac{m_2}{m}\left[\frac{9}{2}\frac{n_1}{\tau_2} + \left(z_1 + \frac{m_2}{m}\frac{T_2 - T}{T_2}z_2\right)\frac{n_2 n_3}{n_2 + n_3}\frac{1}{\tau_{23}}\right],$$

$$c_{22}^{11} = \frac{3}{2}\frac{n_1}{\tau_1} + \frac{z_3}{\tau_{13}}\frac{n_1 n_3}{n_1 + n_3} + \frac{15}{2}\frac{n_1}{\tau_2}\frac{m_2}{m}, \quad c_{22}^{12} = c_{22}^{32} = c_{22}^{21} = c_{22}^{23} = 0,$$

$$c_{22}^{13} = c_{22}^{31} = -\frac{z_4}{\tau_{13}}\frac{n_1 n_3}{n_1 + n_3}, \tag{6}$$

$$c_{22}^{33} = \frac{1}{2}\frac{n_3}{\tau_3} + \frac{z_3}{\tau_{13}}\frac{n_1 n_3}{n_1 + n_3} + \frac{z_7}{\tau_{23}}\frac{n_2 n_3}{n_2 + n_3}\frac{m_2}{m},$$

$$c_{22}^{22} = \frac{3\sqrt{2}}{4}\frac{n_2}{\tau_2} + 3\left(\frac{69}{16}\frac{T}{T_2} - \frac{17}{16}\right)\frac{n_1}{\tau_2}\frac{m_2}{m} + \left(z_5\frac{T}{T_2} - z_6\frac{T_2 - T}{T_2}\right)\frac{m_2}{m}\frac{n_2 n_3}{n_2 + n_3}\frac{1}{\tau_{23}}; \tag{6 (cont'd)}$$

$$\tau_1 = \frac{3\sqrt{m}(kT)^{3/2}}{4\sqrt{\pi}\lambda_1 e^4 n_1}, \qquad \tau_3 = \frac{1}{8\sqrt{\pi}\Omega^{(2,2)*}\sigma^2}\left(\frac{m}{kT}\right)^{1/2}\frac{1}{n_3}$$

$$\tau_2 = \frac{3\sqrt{m_2}(kT_2)^{3/2}}{4\sqrt{2\pi}\lambda_2 e^4 n_2}, \qquad \lambda_\alpha = \frac{1}{2}\ln\frac{k^3 T_\alpha^2 T T_2}{\pi e^6 (n_1 T + n_2 T_2)} \quad (\alpha = 1, 2). \tag{7}$$

Here the mass of the ions and neutrals is considered to be the same, i.e., $m = m_1 = m_3$, where e is the absolute magnitude of the electron charge, k is the Boltzmann constant. The interaction between the charged particles is of the coulomb type, while for the interaction between neutrals Lennard-Jones potential is assumed. Calculations for this potential and tables for $\Omega^{(2,2^*)}$ and σ are given in [4]. We also have

$$\frac{z_1}{(n_2 + n_3)\tau_{23}} = 16\left[\frac{5}{2}\Omega_{23}^{(1)}(1) - \Omega_{23}^{(1)}(2)\right],$$

$$\frac{z_2}{(n_2 + n_3)\tau_{23}} = 16\left[2\Omega_{23}^{(1)}(2) - \Omega_{23}^{(2)}(2)\right],$$

$$\frac{z_3}{(n_1 + n_3)\tau_{13}} = 8\left[\frac{35}{32}\Omega_{13}^{(1)}(1) - \frac{5}{8}\Omega_{13}^{(1)}(2) + \frac{1}{8}\Omega_{13}^{(1)}(3) + \frac{1}{8}\Omega_{13}^{(2)}(3)\right],$$

$$\frac{z_4}{(n_1 + n_3)\tau_{13}} = 8\left[\frac{35}{32}\Omega_{13}^{(1)}(1) - \frac{5}{8}\Omega_{13}^{(1)}(2) + \frac{1}{8}\Omega_{13}^{(1)}(3) - \frac{1}{8}\Omega_{13}^{(2)}(3)\right], \tag{8}$$

$$\frac{z_6}{(n_2 + n_3)\tau_{23}} = 16\left[\frac{75}{16}\Omega_{23}^{(1)}(1) - \frac{65}{8}\Omega_{23}^{(1)}(2) + \frac{15}{4}\Omega_{23}^{(1)}(3) - \frac{1}{2}\Omega_{23}^{(1)}(4)\right],$$

$$\frac{z_5}{(n_2 + n_3)\tau_{23}} = 16\left[\frac{25}{4}\Omega_{23}^{(1)}(1) - 5\Omega_{23}^{(1)}(2) + \Omega_{23}^{(1)}(3)\right],$$

$$\frac{z_7}{(n_2 + n_3)\tau_{23}} = 40\Omega_{23}^{(1)}(1).$$

Integrals for $\Omega_{\alpha\beta}^{(l)}(p)$ are given in [3]. In (5) quantities $\sim (m_2/m)$ in comparison with unity are neglected, and in (6) quantities $\sim (m_2/m)^2$. If interaction between neutrals and charged particles is taken to be Maxwellian, then

$$\tau_{13} = \frac{\sqrt{m}}{\sqrt{\varphi_1}(n_1 + n_3)}, \qquad \tau_{23} = \frac{\sqrt{m_2}}{\sqrt{\varphi_2}(n_2 + n_3)} \tag{9}$$

$$z_1 = 0, \quad z_2 = 19.2, \quad z_3 = 5.35, \quad z_4 = 1.7, \quad z_5 = -z_6 = z_7 = 19.9.$$

The quantities φ_1 and φ_2 are estimated in [3]. It follows from (4)-(9) that the law of interaction between electrons and neutrals has a strong effect on the magnitude and sign of the isotropic corrections. Using the definition of the energy exchange rate $Q_2 = -Q_1 - Q_3$ given in [3] and relations (1)-(3), we obtain

$$Q_2 = Q_{21} + Q_{23} \tag{10}$$

$$Q_{2\alpha}^1 = Q_{2\alpha}^0 + 16\frac{m_2}{m}g_{22}kT\left\{\frac{5}{2}\Omega_{2\alpha}^{(1)}(1) - \Omega_{2\alpha}^{(1)}(2) -\right.$$

$$-\frac{T_2-T}{T}\left[\frac{15}{8}\,\Omega_{\alpha 2}{}^{(1)}(1)-\frac{5}{2}\,\Omega_{\alpha 2}^{(1)}(2)+\frac{1}{2}\,\Omega_{\alpha 2}{}^{(1)}(3)\right]\Big\}\,n_2 n_\alpha$$

$$(\alpha = 1, 3);$$

taking into account the first correction to the isotropic distribution function and neglecting quantities $\sim (m_2/m)^3$

$$Q_{\alpha\beta}{}^0 = 16 n_\alpha n_\beta \frac{m_\alpha m_\beta}{(m_\alpha + m_\beta)^2} k (T_\alpha - T_\beta)\,\Omega_{\alpha\beta}{}^{(1)}(1). \qquad (11)$$

A general expression for $Q^\circ_{\alpha\beta}$ was obtained in [5] in different form. For the elastic sphere model, the results of [5] coincide with (11), while for Coulomb interaction (11) coincides with the result of [6]. For a completely ionized plasma, we have from (10) and (11)

$$Q_{21}{}^1 = \frac{3 n_1 k (T_2 - T)}{\tau_2}\,\frac{m_2}{m}\left\{1-\left[\frac{m_2}{m}\,\frac{T}{T_2}\,\frac{45}{4\sqrt{2}}\left(1-\frac{1}{5}\,\frac{T_2}{T}\right)\frac{n_1}{n_2}\right]\right\}\times$$

$$\times\left\{\left[1+\left(\frac{69}{4\sqrt{2}}\,\frac{T}{T_2}-\frac{17}{4\sqrt{2}}\right)\frac{n_1 m_2}{n_2 m}\right]\right\}^{-1}. \qquad (12)$$

Integrals for $\Omega_{\alpha\beta}^{(p)}(q)$ for Coulomb interaction are evaluated [2]. For a quasi-neutral plasma, expression (12) is in good agreement with the more accurate result obtained in [1]. Instead of the complex function $\Psi(x)$ determined in [1], we have the function

$$\varphi(x) = \frac{1}{1 + (69/4\sqrt{2}\,x)}, \qquad x = \frac{m T_2}{m_2 T}. \qquad (13)$$

The difference between the values of the functions $\varphi(x)$ and $\Psi(x)$ is clear from the table given below:

$x = 10$	20	30	40	100	∞
$\psi = 0.32$	0.52	0.62	0.7	0.9	1
$\varphi = 0.45$	0.63	0.71	0.77	0.9	1.

We note that in the hydrodynamic approximation, the magnetic field does not affect the isotropic correction.

For Coulomb interaction with $m_\alpha/m_\beta \ll 1$, expression (11) reduces to the Landau formula [7]. The result in [5] for Coulomb intereaction does not agree with [6, 7]. This was pointed out in [8], where a rough estimate was obtained for the general expression for the energy exchange rate for multitemperature Maxwell distributions.

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